ISCE Training Note No. 3.2

Resistors, inductors and capacitors

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1 Resistors in d.c. and a.c. circuits

Until we get to radio frequencies, we can usually assume that resistors behave in the same way for a.c. as for d.c.. However, that's not the whole story. Ohm's Law makes an assumption - that the temperature of the resistance element stays constant. But, because there is power dissipated in it, it doesn't stay constant. For pure metals, the **resistivity** changes with temperature, usually increasing above room temperature. The effect is small but not negligible – for copper it is 0.4% per kelvin. For this reason, resistors that aren't intended to be temperature-sensitive are made from alloys or metal oxides whose resistivity does not vary very much with temperature.

Practical resistors also won't stand unlimited voltage across them, even if the current is very small because the resistance value is high. Designers brought up on transistor circuits have often forgotten this when it comes to things like switch-mode power supplies, where there may be 300 V or even more across a resistor, maybe rather an important resistor, such as the 'start-up', which provides temporary power to the control electronics to start the switching action. Most resistors have a limit of 250 V, but there are also special types for safety-critical applications that are rated at several kilovolts (Philips VR25 and VR37, for example).

2 Inductors

Designers have traditionally been reluctant to use inductors unless it was unavoidable, because there were no 'off-the-shelf' standard ranges: you had to design it yourself and get somebody to wind it. But now there are standard ranges – many of them, and all you may need to do is to choose the appropriate type. It's different from, but not much more difficult than, deciding that a 10 W wirewound resistor is not the best for a microphone input bias resistor, or a 10 nF polystyrene capacitor for mounting next to a heating element.

For air-cored inductors, there are three main characteristics: the inductance, the d.c. resistance and the maximum permissible current. For inductors with magnetic cores, you also need to consider that the inductance and the core losses are frequency and voltage dependent, so you need to know the values at or near the same conditions as you intend to use the component. The effects of all losses are often expressed as a 'Q' value, a higher Q indicating lower losses. Note, however, that Q varies a great deal with frequency and often with applied voltage.

3 Capacitors

These are often regarded as a bit mysterious, because they don't pass d.c., and it's not clear how a.c. can 'get through' the insulating 'dielectric' between the electrodes. The answer is that it doesn't: a capacitor is a sort of storage jar (the first ones really were jars, and the Royal Navy used the 'jar' as a unit of capacitance into the 20th century; it was about 11 nF) and the electrons that form the current simply flow into it and then flow back out again.

Each electron carries a tiny electric charge, and charge Q is measured in coulombs (symbol

C), 1 coulomb is the charge transferred by 1 A flowing for 1 s. The value of a capacitor is measured in farads, symbol F. The farad, coulomb and volt are related by definition: a capacitor of 1 F has 1 V across it when it has 1 C stored in it. In general, we have another equation like Ohm's law: this one is called Faraday's law:

$$Q = CU$$

There are many different types of capacitor, and more than one type may be suitable for a given application. The most important characteristics are the value and tolerance, and the voltage rating. Capacitors also have losses: for low frequencies, this is now often expressed as 'equivalent series resistance' (ESR). For higher frequencies, capacitor losses are expressed in terms of Q values, which are generally very high, or as 'loss tangent' or 'tanδ' values, which are very close to 1/Q.

4 Series and parallel connections

4.1 Resistors

We have already quietly assumed that resistors in series add their values: it seems obvious that if we have two pieces of wire, each 10 m long and with a resistance of 1 Ω, and we join them up to make a 20 m length, its resistance must be 2 Ω.

Now suppose we join them in parallel instead, to make a wire 10 m long but twice as thick. It doesn't take much reasoning to deduce that the resistance is 5 Ω now. Now suppose we join a 20 m length (2 Ω) and a 10 m length (1 Ω) in parallel. What is the resulting resistance?

Let us connect a 1 V supply across the wires. The 2 Ω resistance will draw 0.5 A and the 1 Ω resistance will draw 1 A, so the total current is 1.5 A. Ohm's Law then tells us that R = U/I = 1/1.5 = 0.667 Ω. Let us look at the steps involved. First, we did $U/R_1 = I_1$ and $U/R_2 = I_2$. Then we added the currents, to get $(U/R_1 + U/R_2)$, and then divided U by this value:

$$R = U/(U/R_1 + U/R_2)$$

Not a nice-looking equation. Let us turn it upside down (formally, 'divide 1 by both sides'):

$$1/R = (U/R_1 + U/R_2)/U$$

$$= (1/R_1 + 1/R_2)$$

We can do this for any number of resistors in parallel: the resulting resistance is the reciprocal ('1 divided by') of the sum of the reciprocals of the individual resistor values.

For two resistors, we can get a more convenient expression for paper and pencil calculation, by manipulating the equation for 1/R above. If we bring the two fractions on the right into terms of a common denominator we get:

$$1/R = (R_1 + R_2)/R_1R_2, \text{ or } R = R_1R_2/(R_1 + R_2)$$

Note that this isn't as convenient on a calculator because of the need to add up after you pressed the 'divide' button.

Now for something you won't find in the textbooks. We often want to find what value resistor to put in parallel with another to make a value we want but haven't got – often a value between two preferred values. The equation for this is:

$$R = R_1R_2/(R_1 - R_2)$$

Clearly, R_1 must be larger than the wanted odd value R_2, because otherwise R would have to be negative. The proof of this equation is left as an exercise.

4.2 Inductors

We hardly ever connect inductors in series or parallel, but if we did, they would behave in the same way as resistors **if their magnetic fields don't interact.** If their fields do interact, **all bets are off**. You have to analyse the magnetic field situation with considerable insight to be able to determine what the result may be.

4.3 Capacitors

Being storage jars for electric charge, it isn't difficult to imagine that connecting them in parallel adds up the individual capacitance values. In series, they behave like **resistors in parallel.**

4.4 Preferred values

Why do we get resistors and capacitors in those strange series of odd values? It's all to do with variations in manufacture, and tolerances. If we buy some 1 kΩ resistors, they won't all be exactly 1000.0000.... Ω. There will be a specified tolerance, usually ±1% these days, although you can get ±0.01% if you are very rich. Now, explaining in terms of ±1% tolerance leads us into the even funnier numbers of the E96 series, which few people need to use, so we will take the tolerance as being ±10%, (which is realistic for some types anyway).

The manufacturing process for 1 kΩ resistors produces a spread of values either side of the target. So do the processes for lower and higher values, and the manufacturer wants to sell all he makes for as good a price as possible. Those resistors between 900 Ω and 1100 Ω can be sold as '1 kΩ ±10%' (brown, black, red, silver in the four-band code, made for people; the five-band code – brown, black, black, brown, silver – is made for Martians or something). Those below 900 Ω or above 1100 Ω could be sold as '1 kΩ ±20%' for a lower price. But if they are within 10% of either a lower or a higher standard value, they could be sold for the higher price. In order to be able to sell everything, the steps in the series need to correspond to twice the tolerance. So, the series for ±10% components goes (nearly) in steps of 20%: 1.0, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 4.7, 5.6, 6.8. 8.2, 10. This is called the E12 series, because there are 12 values per decade. To get truly equally-spaced values, the ratio between adjacent values should be $10^{1/12}$ = 1.2115..., but that clearly gives values that can't be expressed in the 4-band or even 5-band colour code.